

# PRERAINBOW OSCILLATIONS IN $^3\text{He}$ SCATTERING FROM THE HOYLE STATE OF $^{12}\text{C}$ AND ALPHA PARTICLE CONDENSATION

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$^3\text{He}+^{12}\text{C}$  scattering is studied in a coupled channel method by using a double folding model with microscopic wave functions of  $^{12}\text{C}$ . Experimental angular distributions in elastic and inelastic scattering to the  $2^+$  (4.44 MeV),  $0_2^+$  (7.65 MeV) and  $3^-$  (9.63 MeV) states of  $^{12}\text{C}$  are well reproduced. It is found that the Airy minimum of the prerainbow oscillations for the Hoyle state is considerably shifted to a larger angle due to its dilute density distribution compared with that of the normal ground state in agreement with the idea of  $\alpha$  particle condensation.

*Keywords:*  $^3\text{He}+^{12}\text{C}$  scattering, nuclear rainbow, prerainbow, alpha particle condensation.

## 1. Introduction

Bose-Einstein condensation (BEC) has been well known in a dilute gas.<sup>1</sup> The macroscopic properties such as superconductivity and superfluidity in both  $^3\text{He}$  and  $^4\text{He}$  are understood in relation to BEC. Recently it has been speculated that the  $0_2^+$  (7.65 MeV) state of  $^{12}\text{C}$ , the Hoyle state, is a Bose-Einstein condensate of three  $\alpha$  particles.<sup>2</sup> Uegaki *et al.*<sup>3</sup> and Kamimura and Fukushima<sup>4</sup> studied the  $\alpha$  cluster structure of  $^{12}\text{C}$  thoroughly in the microscopic cluster model and showed that the  $0_2^+$  state of  $^{12}\text{C}$ , has a loosely coupled three  $\alpha$  cluster structure with an  $\alpha\otimes^8\text{Be}$  configuration. Recently it has been shown that the wave functions of Uegaki *et al.*<sup>3</sup> and Kamimura and Fukushima<sup>4</sup> are almost completely equivalent to the wave function of an  $\alpha$  particle condensate that the three  $\alpha$  particles are sitting in the lowest  $0s$  state in a dilute gas.

A macroscopic property that is peculiar to BEC such as superconductivity and superfluidity has not been observed in the case of  $\alpha$  particle condensation. Recently Kokalova *et al.*<sup>5</sup> proposed a new experimental way of testing BEC of  $\alpha$  particles

in nuclei by directly observing the enhancement of  $\alpha$  particle emission and the multiplicity partition of the possible emitted  $\alpha$  particles. It is important to find a phenomenon which strongly reflects the properties of Bose-Einstein condensation of  $\alpha$  particles.

We show that the dilute property of the matter density due to Bose-Einstein condensation can be seen in the nuclear refractive phenomena. Nuclear rainbow scattering has been powerful in the study of nucleus-nucleus interaction when absorption is incomplete.<sup>6</sup> It has been shown that rainbow scattering and the evolution of the Airy minimum can also be seen in inelastic scattering.<sup>7</sup> It is expected that the refractive effect becomes much larger and can be seen clearly at low incident energies. Recently a new concept of prerainbow has been proposed at the lower energy region.<sup>8</sup> The refractive index  $n$  is related to the optical potential  $V$  as follows:

$$n(r) = \sqrt{1 - \frac{V(r)}{E_{c.m.}}}. \quad (1)$$

There is no useful experimental data in inelastic  $\alpha$  particle scattering to the Hoyle state in the low energy region. Fortunately, for  ${}^3\text{He}$  scattering from  ${}^{12}\text{C}$  there is an experimental angular distribution at 34.7 MeV measured by Fujisawa *et al.*,<sup>9</sup> which had been unnoticed for many years and to which no theoretical attention from the viewpoint of  $\alpha$  particle condensation in the Hoyle state had been paid.

## 2. The double folding model

We study elastic and inelastic  ${}^3\text{He}+{}^{12}\text{C}$  scattering in the microscopic coupled channel method by taking into account simultaneously the  $0_1^+$  (0.0 MeV),  $2^+$  (4.44 MeV),  $0_2^+$  (7.65 MeV), and  $3^-$  (9.63 MeV) states of  ${}^{12}\text{C}$ . The diagonal and coupling potentials for the  ${}^3\text{He}+{}^{12}\text{C}$  system are calculated with the double folding model:

$$V_{ij}(\mathbf{R}) = \int \rho_{00}^{({}^3\text{He})}(\mathbf{r}_1) \rho_{ij}^{({}^{12}\text{C})}(\mathbf{r}_2) v_{\text{NN}}(E, \rho, \mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2, \quad (2)$$

where  $\rho_{00}^{({}^3\text{He})}(\mathbf{r})$  is the ground state density of  ${}^3\text{He}$  taken from Cook *et al.*,<sup>10</sup> while  $v_{\text{NN}}$  denotes the density-dependent M3Y effective interaction (DDM3Y).<sup>11</sup>  $\rho_{ij}^{({}^{12}\text{C})}(\mathbf{r})$  represents the diagonal ( $i = j$ ) or transition ( $i \neq j$ ) nucleon density of  ${}^{12}\text{C}$  calculated in the resonating group method by Kamimura *et al.*<sup>4</sup>

The folding potential is very sensitive to the wave functions used, which serves as a good test of the validity of the wave function. In the analysis we introduce the normalization factor  $N_R$  for the real part of the potential and phenomenological imaginary potentials with a Wood-Saxon form factor (volume absorption) and a derivative of the Wood-Saxon form factor (surface absorption) for each channel.

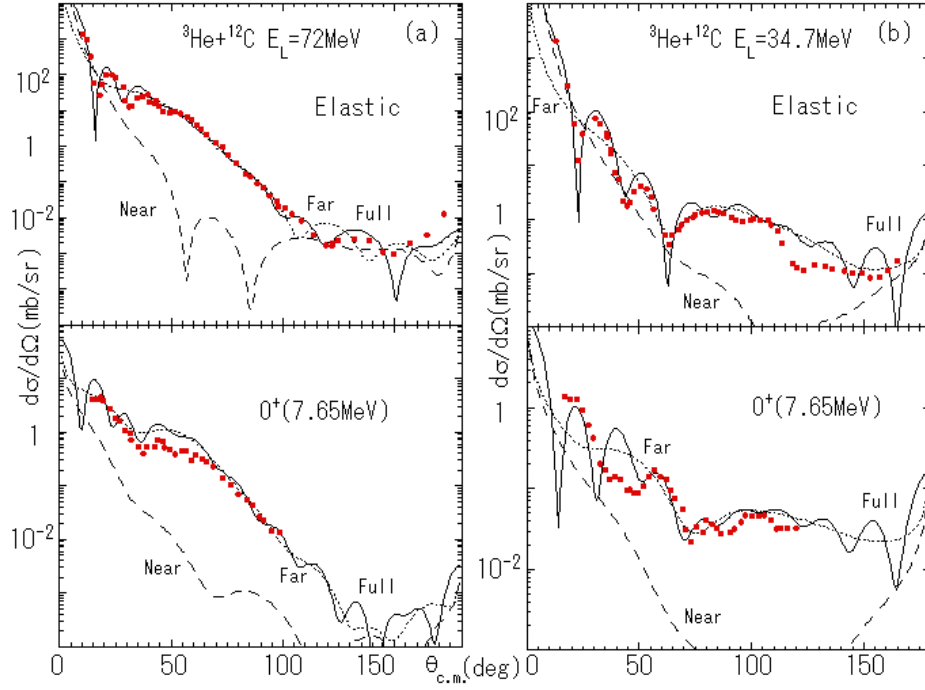


Fig. 1. The calculated angular distributions (solid lines) in  ${}^3\text{He}+{}^{12}\text{C}$  scattering at  $E_L=34.7$  and 72 MeV are decomposed into farside (dotted lines) and nearside (dashed lines) components and compared with the experimental data (points).<sup>9,13</sup>

### 3. Analysis of refractive ${}^3\text{He}+{}^{12}\text{C}$ scattering

In Fig. 1 angular distributions calculated using a coupled channel method at  $E_L=34.7$  MeV and 72 MeV are compared with the experimental data. The calculation reproduces the experimental angular distributions for the ground state and the Hoyle state as well as the  $2^+$  and  $3^-$  states.<sup>14</sup>

By decomposing the calculated scattering amplitude into farside and nearside contributions, the Airy minimum of the rainbow at 72 MeV and the prerainbow oscillations at 34.7 MeV for the Hoyle state is identified. At  $E_L=72$  MeV the first Airy minimum  $A1$  appears at  $35^\circ$  for the  $0_2^+$  state. For elastic scattering a clear minimum is not seen in the angular distribution of the farside cross sections and the Airy minimum in the experimental data is obscured by the interference between the farside and nearside amplitudes. On the other hand, the  $A1$  minimum for the  $0_2^+$  state is clearly seen in the farside cross sections because the minimum is shifted to a larger angle where the nearside contribution is much smaller.

The situation is more clearly seen in the Airy structure at low the incident energy region where no typical rainbow falloff of the dark side appears. At  $E_L=34.7$  MeV in Fig. 1 the Airy minimum  $A1$  appears at  $60^\circ$  for elastic scattering and  $75^\circ$  for

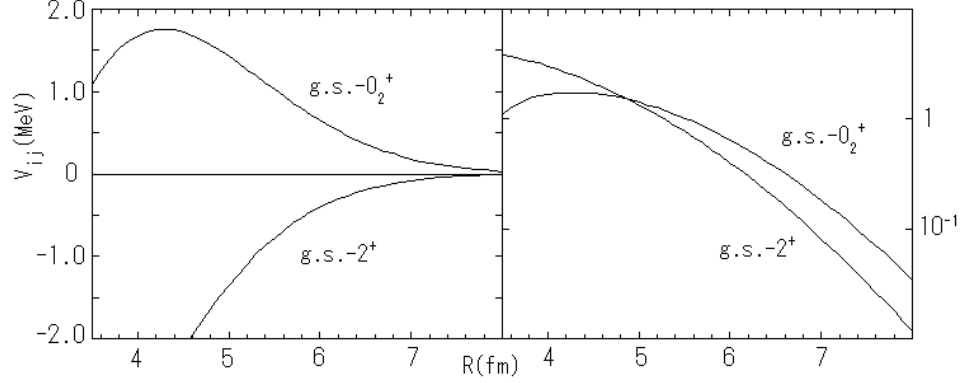


Fig. 2. Coupling potentials for the g.s.- $2_1^+$  and g.s.- $0_2^+$  channels in  ${}^3\text{He}+{}^{12}\text{C}$  scattering at  $E_L=34.7$  MeV are shown on a linear scale (left) and their absolute values are shown on a logarithmic scale (right) to emphasize the difference at the surface region.

the  $0_2^+$  state. The latter is much shifted to a larger angle and the Airy minimum is not at all obscured by the nearside contributions. For the  $0_2^+$  state at  $E_L=34.7$  the nearside contributions are much smaller than the farside contributions compared with the elastic scattering case.

Thus the difference of the refraction between the ground state and the  $0_2^+$  state is much more clearly seen in the prerainbow oscillations at 34.7 MeV than in the rainbow at 72 MeV. This shows that the incident  ${}^3\text{He}$  is strongly refracted in the Hoyle state in accordance with the previous finding in  $\alpha+{}^{12}\text{C}$  scattering<sup>15</sup> that the state has a large lens composed of three  $\alpha$  particles in a dilute density distribution.

In Fig. 2 the coupling potentials for the g.s.- $2_1^+$  and g.s.- $0_2^+$  channels are compared. It is clear that the coupling potential for the g.s.- $0_2^+$  channel is considerably extended to the outer region, which enhances the inelastic scattering to the Hoyle state in the surface region.

In fact, in Fig. 3 we see that even at the low energy region where so many high partial waves are not involved, inelastic scattering to the Hoyle state occurs at large angular momenta (that is, large radius) compared with that to the normal g.s. and  $2_1^+$  states. This becomes clearer at the high energy region as was discussed in  $\alpha$  particle scattering from  ${}^{12}\text{C}$  at  $E_L=139$  MeV.<sup>16</sup> These facts are in accordance with the previous finding that the Hoyle state has a large radius compared with the normal ground state.

#### 4. Summary

We have shown that the prerainbow oscillation is useful for studying the dilute density distribution due to Bose-Einstein condensation of  $\alpha$  particles. The present approach is applicable not only to  ${}^3\text{He}$  scattering but also to heavy ion rainbow scattering. The  ${}^{16}\text{O}+{}^{12}\text{C}$  rainbow and prerainbow scattering will be useful to re-

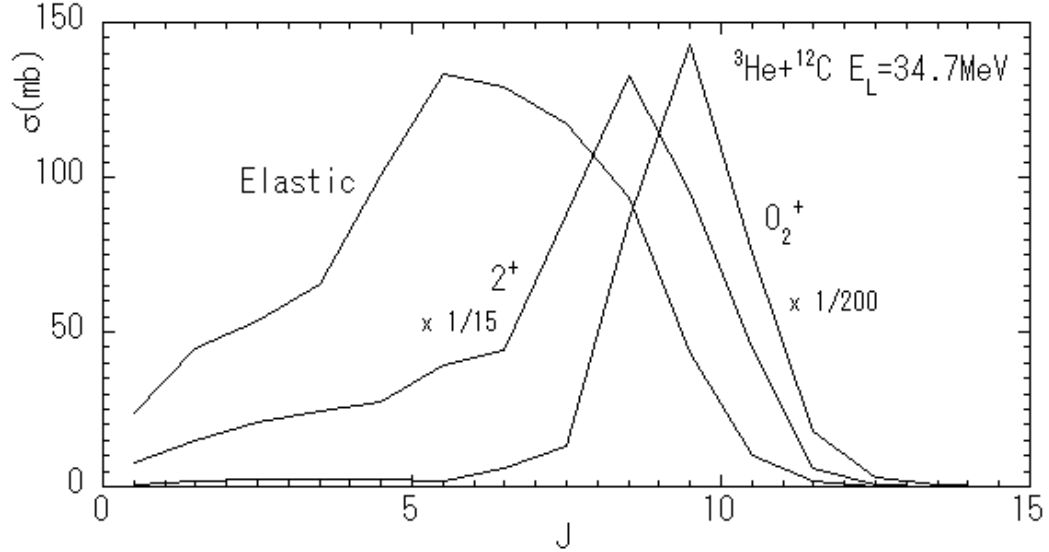


Fig. 3. Calculated partial cross sections for elastic and inelastic  $^3\text{He}$  scattering to the  $2_1^+$  and  $0_2^+$  state of  $^{12}\text{C}$  at  $E_L=34.7$  MeV are shown as a function of total angular momentum.

confirm the present conclusions because the refractive effect is very strong and clear Airy minima of higher order can be expected.

It has been also suggested that the  $\frac{1}{2}^-$  (8.86 MeV) state in  $^{13}\text{C}$ , the  $0^+$  (9.746 MeV) state in  $^{14}\text{C}$  and the  $0^+$  ( $\sim 29$  MeV) state in  $^{16}\text{C}$  may be a state with one, two and four additional neutrons to the  $0_2^+$  state of  $^{12}\text{C}$ . If the above states have a dilute density distribution, the prerinbow oscillations for these states would be considerably different from a state with a normal density distribution. This kind of experiment is highly desired.

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